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This booklet contains the list of participants and the list of abstracts, collected by the Alfréd Rényi Institute of Mathematics. If you prefer to remove your data from the list of participants, or list of abstracts, please contact us in email until August 13, 2019.

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On the generalized Cesáro summability of trigonometric Fourier series

Teimuraz Akhobadze

In [Akh00] is introduced the notion of $BV(p(n) \uparrow p, \varphi)$ class of functions of bounded variation. If $\varphi(n) = 2^n$ then this class coincides with the class $BV(p(n) \uparrow p)$, introduced by Kita and Yoneda [KY90]. In [Akh02] the notion of the class of functions $B\Lambda(p(n) \uparrow \infty, \varphi)$ is introduced. There are studied classes $B\Lambda(p(n) \uparrow \infty, \varphi)$ and, in particular, it is proved that this class is wider then the class of functions $BV(p(n) \uparrow \infty, \varphi)$ even in the case of space of continuous functions.

It is known that the order of the n-th coefficients of trigonometric Fourier series of a function of bounded variation is $O(1/n)$. Siddiqi [Sid72] proved that the order of the Fourier coefficients of functions in BV_p ($p > 1$) is $O(1/n^{1/p})$. On the other hand it's known (see [Zyg02, Chapter III, (1.22)]) that if a number series $\sum u_k$ is summable by the method (C, α), $\alpha > -1$, then $u_n = o(n^\alpha)$. This means that for the summability by method $(C, -\alpha)$, $\alpha > 0$, of Fourier series of the class BV_p it is necessary that $\alpha p < 1$. So, for the classes $BV(p(n) \uparrow \infty, \varphi)$ trigonometric Fourier series can not be $(C, -\alpha)$ summable for any positive α. Above reasoning motivate us to consider generalized Cesáro $(C, -\alpha_n)$ method (introduced by Kaplan [Kap60]) for the summability of Fourier trigonometric series of functions from the class $B\Lambda(p(n) \uparrow p, \varphi)$.

In this report we are going to give the estimate, by the norm of L^r, of the deviation $||\sigma_n^{-\alpha_n}(f, \cdot) - f(\cdot)||_{L^r}$, $1 \le r \le +\infty$ $(0 < \alpha_n < 1)$, in terms of modulus of continuity and of generalized bounded variation.

This is a joint research with Sh. Zviadadze (TSU).

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Optimal recovery of a derivative of an analytic function from values of the function given with an error on a part of the boundary

Roman Akopyan

A complete solution of several related extremal problems for functions analytic in a simply connected domain G with a rectifiable Jordan boundary Γ will be presented. In particular, the problem of optimal recovery of a derivative at a point $z_0 \in G$ from limit boundary values given with an error on a measurable part γ_1 of the boundary Γ for the class Q of functions with limit boundary values bounded by 1 on $\gamma_0 = \Gamma \setminus \gamma_1$ as well as the problem of the best approximation of the derivative at a point $z_0 \in G$ by bounded linear functionals in $L^\infty(\gamma_1)$ on the class Q.

One of the problems under consideration is the problem of optimal recovery of the value of a derivative at a point z_0 of a function analytic in the domain G from boundary values of the function on γ_1 given with an error δ with respect to the norm of $L^\infty(\gamma_1)$ and from the additional information that the function belongs to the class Q. More precisely, for an unknown function f from the class Q, let a function $q \in L^\infty(\gamma_1)$ be given such that $\|f - q\|_{L^\infty(\gamma_1)} \le \delta$. We aim to recover the value of the derivative $f'(z_0)$, $z_0 \in G$, from q in the best possible (optimal) method. For the set $\mathcal{R}$ of methods of recovery, among which the optimal one is chosen, we take either the set $\mathcal{F}$ of all possible functionals on $L^\infty(\gamma_1)$ or the set $\mathcal{B}$ of all bounded functionals or the set $\mathcal{L}$ of all linear functionals. The exact statement of the problem is as follows. For a number $\delta \ge 0$ and a recovery method $T \in \mathcal{R}$, the value

$$\mathcal{U}(T, \delta) := \sup \left\{ |f'(z_0) - Tq| \colon f \in Q,\, q \in L^\infty(\gamma_1),\, \|f - q\|_{L^\infty(\gamma_1)} \le \delta \right\}$$

is the error of recovering a value of the derivative of a function from the class Q at the point z_0 from boundary values of the function on γ_1 given with error δ with respect to the norm $L^\infty(\gamma_1)$. Then

$$\mathcal{E}_{\mathcal{R}}(\delta) := \inf \{ \mathcal{U}(T, \delta) \colon T \in \mathcal{R} \} \tag{1}$$

is *the value of optimal recovery* of a value of the derivative at the point z_0 (or, equivalently, the value of optimal recovery of the functional $\Upsilon^1_{z_0}$) by means of recovery methods $\mathcal{R}$ on functions from Q from their boundary values on γ_1 given with error δ. The problem consists in calculating the value $\mathcal{E}_{\mathcal{R}}(\delta)$ and finding an optimal recovery method, which is a functional at which the infimum in (1) is attained.

Discussions of statements of the problems, their relation and previous results can be found in [Ako18].

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Bounds for the sharp constant in Jackson–Nikol'skii inequality between the uniform norm and integral norm of trigonometric polynomials

Aleksandr Babenko and Marina Deikalova

The talk will contain the results improving known two-sided bounds for the sharp constant in the Jackson–Nikol'skii inequality between the uniform norm and integral norm of trigonometric polynomials obtained by L.V. Taikov and D.V. Gorbachev earlier. We extend an approach proposed by Ya.L. Geronimus and F. Peherstorfer to characterize canonical sets of points in the space of trigonometric polynomials on the period in the case of the unit weight to the case of a special Jacobi weight.

This is a joint research with D.V. Gorbachev (Tula State University) and Yu.V. Kryakin (University of Wroclaw).

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On the constant in Steinitz's theorem

Imre Bárány

Let $\|.\|$ be a norm in $\mathbf{R}^d$ whose unit ball is B. Assume that $V \subset B$ is a finite set of cardinality n, with $\sum_{v\in V} v = 0$. A famous theorem of Steinitz from 1914 says that, under these conditions there is an ordering $\{v_1, \dots, v_n\}$ of the elements of V such that $\| \sum_1^k v_i\| \le 2d$ for every $k \in [n]$. The question whether the constant $2d$ can be significantly improved for some specific norms has been wide open. In fact no norm is known for which the constant is $o(d)$. We show that, for the so called *quasi-orderings* of V it can be improved to $O(\sqrt{d})$ for the Euclidean and the max norms. Consequences and further applications will also be given.

This is joint work with Gergely Ambrus and Victor Grinberg.

Delsarte's extremal problem and packing on locally compact Abelian groups

Elena E. Berdysheva

Let G be a locally compact Abelian group, and let Ω_+, Ω_- be two open sets in G. We investigate the constant $\mathcal{C}(\Omega_+, \Omega_-) = \sup\left\{\int_G f : f \in \mathcal{F}(\Omega_+, \Omega_-)\right\}$, where $\mathcal{F}(\Omega_+, \Omega_-)$ is the class of positive definite functions f on G such that $f(0) = 1$, the positive part f_+ of f is supported in Ω_+, and its negative part f_- is supported in Ω_-. In the case when $\Omega_+ = \Omega_- =: \Omega$, the problem is exactly the so-called Turán problem for the set Ω. When $\Omega_- = G$, i.e., there is a restriction only on the set of positivity of f, we obtain the Delsarte problem. The Delsarte problem in $\mathbb{R}^d$ is the sharpest Fourier analytic tool to study packing density by translates of a given "master copy" set, which was studied first in connection with packing densities of Euclidean balls.

We give an upper estimate of the constant $\mathcal{C}(\Omega_+, \Omega_-)$ in the situation when the set Ω_+ satisfies a certain packing type condition. This estimate is given in terms of the asymptotic uniform upper density of sets in locally compact Abelian groups.

This is a joint research with Szilárd Gy. Révész (Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences).

Random walks on the circle and Diophantine approximation

István Berkes

Let $X_1, X_2, \ldots$ be i.i.d. integer-valued random variables, $S_k = \sum_{j=1}^{k} X_j$, let α be an irrational number and let $Z_k = \{S_k \alpha\}$, where $\{\cdot\}$ denotes fractional part. Then Z_k, $k = 1, 2, \ldots$ is a random walk on the circle, and by classical results of probability theory, the distribution of $\{S_k \alpha\}$ converges weakly to the uniform distribution. We say that α has strong Diophantine type $\gamma \geq 1$ if $0 < \liminf_{q \to \infty} q^{\gamma} \|q\alpha\| < \infty$, where $\|\cdot\|$ denotes distance from the nearest integer. Assuming that X_1 has finite variance or heavy tails $\mathbb{P}(|X_1| > x) \sim cx^{-\beta}$ $(0 < \beta < 2)$, we show, letting $\beta = 2$ in the finite variance case, that

$$\psi_\alpha(n) := \sup_{0 \leq x \leq 1} |\mathbb{P}(\{S_k \alpha\} \leq x) - x| = O(n^{-1/(\beta\gamma)})$$

and this estimate is sharp. We also show that the order of magnitude of the discrepancy $D_N(S_k \alpha)$ of the sequence $\{S_k \alpha\}$ suddenly changes when the Diophantine rank γ of α passes through the critical value $\gamma = 2/\beta$.

We also determine, without any assumption on the type of distribution of X_1 and for any $\alpha \neq 0$, the limit distribution of $\sqrt{N} D_N(S_k \alpha\}$ and prove a law of the iterated logarithm for $D_N(S_k \alpha\}$ under the assumption $\psi_\alpha(n) \ll n^{-(1+\delta)}$, $\delta > 0$. This covers the case of absolutely continuous X_1, as well as the integer valued heavy tailed case for $\gamma < 1/\beta$. Assuming only $X_1 > 0$ and $EX_1 < \infty$, but without any tail asymptotics for X_1 in the integer valued case, we prove the central limit theorem and the law of the iterated logarithm for the discrepancy of $\{S_k \alpha)$ with respect to a fixed interval.

Joint work with Bence Borda.

Uncertainty inequalities in finite planes

András Bíró

The uncertainty principle asserts that a nonzero function and its Fourier transform cannot be both highly concentrated on small sets. In this talk we consider this principle on some special finite Abelian groups.

The most known general uncertainty principle states that if G is a finite Abelian group and $f : G \to \mathbf{C}$ is a nonzero function, ϕ is its Fourier transform, then $|\mathrm{supp} f| \cdot |\mathrm{supp} \phi| \geq |G|$. This can be significantly improved for groups of prime order, the following statement was proved independently by Bíró and Tao: if p is a prime, $G = \mathbf{Z}_p$ (where $\mathbf{Z}_p = \mathbf{Z}/p\mathbf{Z}$), and f and ϕ are as above, then we have $|\mathrm{supp} f| + |\mathrm{supp} \phi| \geq p + 1$. This is the best possible result for groups of prime order, however, for a general finite Abelian group the best possible result is not known. In this talk we consider the case of rank 2 elementary Abelian groups, and we sharpen the previously known results.

This is a joint research with V.F. Lev (The University of Haifa at Oranim).

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Remarks on the prime geodesic theorem in 2D and 3D

Giacomo Cherubini

In analogy with the theory of prime numbers, it is possible to consider "prime matrices" in a given matrix group Γ, and the counting function $\pi_\Gamma(x)$ associated to them. For the primes, having an asymptotic with an error term of size $O(x^{1/2+\epsilon})$ is equivalent to the Riemann hypothesis. I will survey the status of the problem for the function $\pi_\Gamma(x)$ in the case when the matrices have integer entries (i.e. $\Gamma = \mathrm{SL}(2,\mathbb{Z})$) or when the entries are Gaussian integers (i.e. $\Gamma = \mathrm{SL}(2,\mathbb{Z}[i])$. The problem has connections to the theory of GL(2) automorphic forms, to Dirichlet L-functions, to class numbers of quadratic number fields, and to the Lindelöf Hypothesis.

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Inequalities on rearrangements of summands with applications in a.s. convergence of functional series

Sergei Chobanyan

Theorem 1. *Let $x_1, \dots, x_n \in X$ be a collection of elements of a real normed space X with $\sum_{i=1}^n x_i = 0$. Then*

a. For any collection of signs $\theta = (\theta_1, \dots, \theta_n)$ there is a permutation $\pi : \{1, \dots, n\} \to \{1, \dots, n\}$ such that

$$\max_{1\le k\le n} \left\| \sum_1^k x_i \right\| + \max_{1\le k\le n} \left\| \sum_1^k \theta_i x_i \right\| \ge 2 \max_{1\le k\le n} \left\| \sum_1^k x_{\pi_\theta(i)} \right\| .$$

The mapping $\theta \to \pi_\theta$ can be written down explicitly.

b. (Transference Theorem) There is a permutation $\sigma : \{1, \dots, n\} \to \{1, \dots, n\}$ such that

$$\max_{1\le k\le n} \left\| \sum_1^k \theta_i x_{\sigma(i)} \right\| \le \max_{1\le k\le n} \left\| \sum_1^k \theta_i \theta_i x_{\sigma(i)} \right\|$$

for any collection of signs $\theta = (\theta_1, \dots, \theta_n)$.

Theorem 1 implies the Maurey–Pisier sign-permutation relationship, Garsia-Nikishin type theorems on rearrangement convergence almost surely of a functional series. It establishes the best constant in the Garsia maximum inequality for rearrangements of orthonormal systems (in the case of a single L_2-function). An assertion similar to Theorem 1 was used by Konyagin and Révész to find conditions under which the Fourier series of a 2π-periodic continuous function f converges a.s. under some rearrangement. Theorem 1 also finds applications in scheduling theory, discrepancy theory and machine learning.

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On some local properties of the conjugate function and the modulus of smoothness of fractional order

Ana Danelia

Moduli of smoothness play a basic role in approximation theory, Fourier analysis and their applications. For a given function f, they essentially measure the structure or smoothness of the function via difference of different orders.

In the theory of real functions there is the well-known theorem of Privalov on the invariance of the Lipschitz classes under the conjugate function $\widetilde{f}$. Analogous problem in terms of modulus of smoothness of fractional order was considered by Samko and Yakubov. They proved that the generalized Holder class is invariant under the operator $\widetilde{f}$.

It is also well-known that Zygmund established that the analogue of Privalov's theorem is valid in terms of modulus of continuity of second order. Afterwards Bari and Stechkin obtained results with behavior of modulus of continuity of higher order of the function f and its conjugate function.

As to the functions of many variables the first result in this directin belongs to Cesari and Zhak. Later there were obtained the sharp estimates for partial moduli of smoothness of different orders in spaces C and L.

In the present report we study the behavior of the smoothness of fractional order of the conjugate functions of many variables at fixed point in the space C if the global smoothness as well as the behavior at this point of the original functions are known. The direct estimates are obtained and exactness of these estimates are established by proper examples.

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Simultaneous approximation by Bernstein polynomials and their integer forms

Borislav R. Draganov

I will present a characterization of the rate of the weighted simultaneous approximation in $L_p[0,1]$, $1 < p \leq \infty$, by the Bernstein operator. As is known, it is defined for $f \in C[0,1]$ by

$$B_n(f,x) := \sum_{k=0}^{n} f\left(\frac{k}{n}\right)\binom{n}{k}x^k(1-x)^{n-k}, \quad x \in [0,1].$$

The weights I consider are those of Jacobi. The characterization of the rate of the simultaneous approximation—a direct inequality and a matching strong two-term converse inequality—is stated by means of appropriate moduli of smoothness or K-functionals. Also, I will state analogous results concerning the Kantorovich operators.

I will also consider the simultaneous approximation by the Bernstein polynomials with integer coefficients in the uniform norm. L. Kantorovich introduced the first such modification. It is given by

$$\widetilde{B}_n(f,x) := \sum_{k=0}^{n} \left[f\left(\frac{k}{n}\right) \binom{n}{k} \right] x^k (1-x)^{n-k},$$

where $[\alpha]$ denotes the largest integer that is less than or equal to the real α. Another integer modification of the Bernstein polynomials, which actually has better approximation properties, is defined by means of the nearest integer. I will give direct and weak converse error estimates for both operators. They are established under quite restrictive assumptions, but they turn out to be necessary too. It is noteworthy that for the derivatives of order two and higher, the necessary conditions for both operators are more restrictive than for the first derivative.

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Zeros of polynomials with restricted coefficients

Tamás Erdélyi

We discuss various recent results on the zeros of polynomials with restricted coefficients. We are especially interested in the number of zeros on the unit circle and the multiplicity of a zero at 1. Let $n_1 < n_2 < \cdots < n_N$ be non-negative integers. In a private communication Brian Conrey asked how fast the number of real zeros of the trigonometric polynomials $T_N(\theta) = \sum_{j=1}^{N} \cos(n_j\theta)$ in a period tends to ∞ as a function N. Conrey's question in general does not appear to be easy. Let $\mathcal{P}_n(S)$ be the set of all algebraic polynomials of degree at most n with each of their coefficients in S. In a large part of this talk we examine the number of unimodular zeros of self-reciprocal polynomials $P \in \mathcal{P}_n(S)$, where $S \subset \mathbb{R}$ is a fixed finite set.

Quasicrystals Fourier and tempered distributions with discrete support or spectrum

Serhii Favorov

Let

$$f_j = \sum_{\lambda \in \Lambda_j,\ k_1+\cdots+k_d \le m(\lambda)} p_{\lambda,k}^{(j)} D^k \delta_\lambda, \quad k \in (\mathbf{N} \cup \{0\})^d, \quad j = 1,2,$$

be tempered distributions on $\mathbf{R}^d$ with discrete supports Λ_j. It can be easily checked that $\sup_{\lambda\in\Lambda_j} m(\lambda) < \infty$.

Theorem 1. *Suppose that* $\inf_{\lambda\in\Lambda_j} \sup_k |p_{\lambda,k}^{(j)}| > 0$ *and the set* $\Lambda_1 - \Lambda_2$ *is discrete. If distributional Fourier transforms* $\hat{f}_j$ *are discrete complex measures with supports* Γ_1, Γ_2 *satisfying the condition*

$$\exists h < \infty,\ c > 0 \quad \text{such that} \quad |\gamma - \gamma'| > c\min\{1, |\gamma|^{-h}\} \quad \forall \gamma, \gamma' \in \Gamma, \tag{*}$$

then Λ_1, Λ_2 *are finite unions of translates of a* ***unique*** *full-rank lattice.*

Theorem 2. *Let* μ *be a discrete complex measure on* $\mathbf{R}^d$ *with a uniformly discrete support* Λ, $|\mu(\{\lambda\})| \ge c > 0$ *for all* $\lambda \in \Lambda$, *and the distributional Fourier transform* $\hat{\mu}$ *be a measure such that* $|\hat{\mu}|(B(r)) = O(r^d)$ *as* $r \to \infty$. *Then* Λ *is a finite union of translates of* ***several*** *full-rank lattices.*

The proofs of these theorems are based on generalization of Wiener's Theorem on Fourier series and properties of almost periodic distributions and measures. In particular, we prove that every tempered distribution whose Fourier transform is a discrete complex measure with support satisfied condition (*) is almost periodic.

Sincov-type functional inequalities and generalized metrics

Włodzimierz Fechner

We will discuss the multiplicative Sincov's inequality:

$$G(x,z) \leq G(x,y) \cdot G(y,z), \quad x,y,z \in X.$$

We assume that X is a topological space and $G\colon X \times X \to \mathbb{R}$ is a continuous map. We also study the reverse inequality:

$$F(x,z) \geq F(x,y) \cdot F(y,z), \quad x,y,z \in X$$

and the additive version of the original inequality (i.e. the triangle inequality):

$$H(x,z) \leq H(x,y) + H(y,z), \quad x,y,z \in X.$$

A corollary for generalized metric is derived.

The talk is based on the following preprint.

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On moment functions on affine groups

Żywilla Fechner

The aim of this talk is to present the form of moment functions on an affine group. Let V be an n-dimensional vector space and let $GL(V)$ denote the general linear group of V. For each subgroup K of $GL(V)$ we form the semidirect product

$$\operatorname{Aff} K = K \ltimes V$$

in the following way: we equip the set $K \times V$ with the following multiplication:

$$(x,u) \cdot (y,v) = (x \cdot y, x \cdot v + u)$$

for each $x,y \in K$ and $u,v \in V$. Let us consider $G = K \ltimes V$ with the compact subgroup $\{(k,0) : k \in K\} \cong K$. Let $g_n\colon G \to \mathbb{C}$ be continuous and K-invariant functions for $n = 0,1,\ldots,N$. The sequence $(g_n)_{n=0,1,\ldots,N}$ is called a generalized K-moment function sequence if

$$\int_K g_n(xky, xk \cdot v + u)\, d\omega(k) = \sum_{j=0}^{n} \binom{n}{j} g_j(x,u) g_{n-j}(y,v) \tag{1}$$

holds for $n = 0,1,\ldots,N$ and for each $(x,u),(y,v) \in G$.

Theorem. *Let V be a finite dimensional vector space and let K be a compact subgroup of $GL(V)$. Then the sequence $(g_n)_{0\le n\le N}$ of K-invariant continuous complex functions on* $\mathrm{Aff}\, K$ *is a generalized moment function sequence if and only if it has the form*

$$g_n(x,u)=\varphi_n(u)\ \ (n=0,1\dots,N) \tag{2}$$

for each (x,u) in $\mathrm{Aff}\, K$, *where $\varphi_n : V\to\mathbb{C}$ is continuous and K-invariant, further we have*

$$\int_K \varphi_n(k\cdot u+v)\,d\omega(k)=\sum_{j=0}^{n}\binom{n}{j}\varphi_j(u)\varphi_{n-j}(v) \tag{3}$$

for $n=0,1\dots,N$ and for each $u,v\in V$.

This is a joint research with László Székelyhidi (University of Debrecen).

The talk is based on the articles cited below.

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The Banach Gelfand triple and Fourier standard spaces

Hans G. Feichtinger

Central objects of *classical Fourier Analysis* are the Fourier transform (often just viewed as an integral transform defined on the Lebesgue space $L^1(R^d)$), convolution operators, periodic and non-periodic functions in L^p-spaces and so on. *Distribution theory* widens the scope by allowing larger families of Banach spaces of functions or generalized functions and extending many of the concepts to this more general setting. Although, according to A. Weil the natural setting for Fourier Analysis (leading to the spirit of Abstract Harmonic Analysis: AHA) most of the time one works in the setting of the Schwartz space $\mathcal{S}(R^d)$ of rapidly decreasing functions and its dual space, the *tempered distributions*. In this setting weighted L^2-spaces and Sobolev spaces correspond to each other in a very natural way.

In this talk we will summarize the advantages with respect to the level of technical sophistication and theoretical background which is possible when one uses instead of the Schwartz-Bruhat space $\mathcal{S}(G)$ the Segal algebra $S_0(G)$ and the resulting Banach Gelfand Triple (S_0, L^2, S_0'), which appears to be suitable for the description of most problems in AHA as well as for many engineering applications (this part is beyond the scope of the current talk). Among others the use of *Wiener amalgam spaces* $W(L^p,\ell^q)$ and *modulation spaces* $M^{p,q}$ (introduced by the author in the 1980s) belong to a comprehensive family of Banach spaces $(B,\|\cdot\|_B)$ embedded between S_0 and S_0', which we call *Fourier Standard Spaces*. These spaces have a *double module structure*, with respect to convolution by L^1-functions and pointwise multiplication with functions from the Fourier algebra FL^1. The most interesting examples are Banach spaces of (generalized) functions containing $S_0(G)$ as a dense subspace and such that time-frequency shifts $f\mapsto\pi(t,\omega)f$ are isometric on $(B,\|\cdot\|_B)$, where

$$\pi(t,\omega)f(x)=e^{2\pi i\omega\cdot x}f(x-t),\quad x,t,\omega\in R^d,$$

or the dual of such a space. There is along list of examples of such spaces.

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Weighted approximation of functions in L_p-norm by Baskakov-Kantorovich operator

Ivan Gadjev

The weighted approximation of functions in L_p-norm by Kantorovich modifications of the classical Baskakov operator is discussed. The weights under consideration are $(1+x)^\alpha, \alpha \in \mathbb{R}$. The weigted error of approximation $\|w(L_nf-f)\|_p$ where L_n is the Kantorovich modification of the classical Baskakov operator is characterized by the next K-functional

$$K_w(f,t)_p = \inf\left\{\|w(f-g)\|_p + t\left\|w\tilde{D}g\right\|_p : f-g \in L_p(w),\ g \in W_p(w)\right\},$$

where

$$\tilde{D} = \frac{d}{dx}\Big(\varphi(x)\frac{d}{dx}\Big),$$

$$\begin{gathered}\varphi(x) = x(1+x),\\ L_p(w) = \{f : wf \in L_p[0,\infty)\}\\ W_p(w) = \big\{f : w\tilde{D}f \in L_p]0,\infty),\ \lim_{x\to 0^+}\varphi(x)f'(x) = 0\big\}, \quad \alpha < 0,\\ W_p(w) = \big\{f : w\tilde{D}f \in L_p]0,\infty),\ \lim_{x\to 0^+,\infty}\varphi(x)f'(x) = 0\big\}, \quad \alpha > 0.\end{gathered}$$

This is a joint research with Parvan Parvanov and Rumen Uluchev (University of Sofia, Bulgaria).

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The Zeros of a Class of Dirichlet Functions

Dorin Ghisa

We study the class of Dirichlet functions obtained as analytic continuation across the line of convergence of Dirichlet series which can be written as Euler products. By using the geometric properties of the mapping realized by these functions, we tackle the problem of the multiplicity of those zeros.

Basis Problem for the spaces of Whitney jets

Alexander Goncharov

The talk consists of two parts. First we show that, for each compact set K of infinite cardinality with an isolated point, the space of Whitney jets on K does not possess a polynomial basis. But then polynomials are dense in any Whitney space. Thus, there are no general results about stability of bases in Fréchet spaces.

Secondly, we discuss possible forms of bases in the spaces of Whitney jets defined on sequences of points.

Uncertainty principles for eventually constant sign bandlimited functions

Dmitry Gorbachev

We study the uncertainty principles related to the generalized Logan problem in $\mathbb{R}^d$. Our main result provides the complete solution of the following problem: for a fixed $m \in \mathbb{Z}_+$, find

$$\sup\{|x|\colon (-1)^m f(x) > 0\} \cdot \sup\{|x|\colon x \in \operatorname{supp} \widehat{f}\,\} \to \inf,$$

where the infimum is taken over all nontrivial positive definite bandlimited functions such that $\int_{\mathbb{R}^d} |x|^{2k} f(x)\, dx = 0$ for $k = 0, \ldots, m-1$ if $m \geq 1$.

We also obtain the uncertainty principle for bandlimited functions related to the recent result by Bourgain, Clozel, and Kahane.

This is a joint research with V. Ivanov (Tula State University, Russia) and S. Tikhonov (CRM, ICREA, Barcelona).

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Sampling, Marcinkiewicz-Zygmund inequalities, approximation, and quadrature rules

Karlheinz Gröchenig

Suppose you are given n samples of a function f on the torus. What can you say about f? How well can you approximate f or the integral $\int_0^1 f(x)dx$? The key notion to answer this question is the notion of Marcinkiewicz-Zygmund (MZ) sets. A MZ-set $X = (X_n)_{n\in\mathbb{N}}$ for the torus is a sequence of finite sets X_n such that

$$A\|p\|_2^2 \leq \sum_{x\in X_n} |p(x)|^2 \leq B\|p\|_2^2 \quad \forall p \in \mathcal{T}_n\,,$$

where $\mathcal{T}_n$ denotes the subspace of trigonometric polynomials of degree n and $A, B > 0$ are two constants *independent* of n.

To produce a good approximation of a smooth function f from its samples $f|_{X_n}$ on an MZ-set, one first constructs a trigonometric polynomial $p_n \in \mathcal{T}_n$ that approximates the given samples optimally. Numerically, this amounts to solving a least squares problem. For f in the Sobolev space H^σ, one can then prove a convergence rate

$$|f - p_n\| = \mathcal{O}(n^{-\sigma+1/2})\,.$$

This also implies an error estimate for quadrature rule that is associated to every MZ-set.

Using the same technique one can prove similar results for functions on a bounded domain in $\mathbb{R}^d$, where the trigonometric system is replaced by the eigenfunctions of the Laplacian, or even more generally on compact Riemannian manifolds.

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The square functional calculus

Bernhard Haak

Square functions and square function estimates are a classical topic and a central tool in harmonic analysis, in particular in the so-called Littlewood-Paley theory. Their history can be traced back to almost a century ago. From the mid 1980's on, the theory of functional calculus for sectorial operators was developed by several people. Building on the seminal works of McIntosh and collaborators a strong link between square function estimates the boundedness of the H^∞-calculus for sectorial operators on (closed subspaces of) L^p was established. Kalton and Weis (in an unpublished and unfortunately never finalized manuscript (2001) then showed how one could pass from L^p-spaces to general Banach spaces. Their manuscript subsequently circulated and inspired a considerable amount of research. In this talk (and our preprint) we structurize and simplify the existing theory in a few basic principles. It turns out that square functions can be seen as a sort of functional calculus for Hilbert space valued functions.

This is joint work with Markus Haase (Kiel).

A Minkowski-type result for linearly independent subsets of ideal lattices

Gergely Harcos

We estimate, in a number field, the maximal number of linearly independent elements with prescribed bounds on their valuations. As a by-product, we obtain new bounds for the successive minima of ideal lattices. Our arguments combine group theory, ramification theory, and the geometry of numbers.

This is joint work with Mikołaj Frączyk and Péter Maga.

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On the growth of Blaschke products for the half-plane

Feliks Hayrapetyan

Let the sequence of complex numbers $\{w_k\}_1^\infty = \{u_k + iv_k\}_1^\infty$ lies in the lower half-plane $G = \{w : \Im(w) < 0\}$ and satisfies the condition $\sum\limits_{k=1}^{\infty} |v_k| < +\infty$. Then the infinite product of Blaschke $B(w) = \prod\limits_{k=1}^{\infty} \frac{w-w_k}{w-\overline{w}_k}$ converges in the lower half-plane G, defining there an analytic function with zeros $\{w_k\}_1^\infty$. We define an integral logarithmic mean of order $q,\ 1 \le q < +\infty$ of Blaschke products for the half-plane by the formula

$$m_q(v,B) = \left(\int_{-\infty}^{+\infty} |\log|B(u+iv)||^q \, du \right)^{\frac{1}{q}}, \qquad -\infty < v < 0.$$

Let's denote by $n(v)$ the number of zeros of the function B in the half-plane $\{w : Im(w) \le v\}$.

In the case of a unit disc, for $q = 2$, the problem about the boundedness of the integraal logarithmic means of Blaschke products was posed by A. Zygmund. In 1969 this problem was solved by the method of Fourier series for meromorphic functions by G.R. MacLane and L.A. Rubel. V.V. Eiko and A.A. Kondratyuk investigated this problem in the general case, when $1 \le q < +\infty$.

In the case of a half-plane the problem of the connection of the boundedness of $m_2(v, B)$ to the distribution of zeros of the products B was solved by the method of Fourier transforms for meromorphic functions.

A.A. Kondratyuk and M.O. Girnik constructed a Blaschke products of given quantity index for the unit disc. They use asymptotic formulas of R.S. Galoyan.

We used the Fourier transforms method for meromorphic functions to characterize the behavior and investigate the growth of the integral logarithmic mean of arbitrary order of Blaschke products for the half-plane. We also proved the existence of Blaschke products of given quantity indexes for the half-plane.

This is a joint research with Mr. G.V. Mikayelyan (PhD, YSU Associate Professor).

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Asymptotics for recurrence coefficients of X1-Jacobi polynomials and Christoffel function

Ágota P. Horváth

Computing asymptotics of the recurrence coefficients of X1-Jacobi polynomials we investigate the limit of Christoffel function. We also study the relation between the normalized counting measure based on the zeros of the modified average characteristic polynomial and the Christoffel function in limit. The proofs of the corresponding theorems with respect to ordinary orthogonal polynomials are based on the three-term recurrence relation. The main point is that exceptional orthogonal polynomials possess at least five-term formulae and so Christoffel-Darboux formula also fails. It seems that these difficulties can be handled in combinatorial way.

On the classes of functions of generalized bounded variation

Koba Ivanadze

The properties of the class of functions of generalized bounded variation is studied. This class $BV^*(f, p_n \uparrow \infty, \phi)$ is complete normed space, it is not separable. The "anomaly" feature of this class is revealed. There is given an example of a function f such that for some $a < x < y$ we have $V(f, p_n \uparrow \infty, \phi, [a, x]) > V(f, p_n \uparrow \infty, \phi, [a, y])$.

There is introduced notion of generalized absolute continuity in the sense of generalized bounded variation. There is given a function which is continuous but not absolute continuous in the sense of generalized bounded variation. A necessary and sufficient condition for f to be generalized absolute continuous is established. The problems of approximation by Steklov's functions and singular integrals are studied.

This is a joint research with T. Akhobadze (TSU).

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Weighted inequalities for the Dunkl-Riesz potential

Valerii I. Ivanov

The Dunkl-Riesz potential I_α^k, $\alpha > 0$, was introduced in 2007 by S. Thangavelu and Y. Xu [TX07] as multiplier operator for which $\mathcal{F}_k(I_\alpha^k f)(y) = |y|^{-\alpha}\mathcal{F}_k(f)(y)$, where $\mathcal{F}_k(f)(y)$ is the Dunkl transform, defined by a root system $R \subset \mathbb{R}^d$, reflection group G generated by R, multiplicity function k on R invariant with respect to G, and Dunkl weight $v_k(x) = \prod_{a\in R} |(a, x)|^{k(a)}$. If $k \equiv 0$, then $\mathcal{F}_k(f)$ coincides with usual Fourier transform.

For the Dunkl-Riesz potential we proved the boundedness conditions in Lebesgue spaces with Dunkl weight and power weights [GIT17], similar to the Hardy–Littlewood–Sobolev–Stein–Weiss conditions for the classical Riesz potential [SW58]. At the conference "Follow-up Approximation Theory and Function Spaces" in the Centre de Recerca Matemática (CRM, Barcelona, 2017) M.L. Goldman raised the question about (L_p, L_q)-boundedness conditions of the Dunkl-Riesz potential with piecewise-power weights. We give an answer to this question. Consideration of piecewise-power weights makes it possible to reveal the influence of the behavior of weights at zero and infinity on the boundedness of the Dunkl-Riesz potential and allows to analyze the necessity of the Hardy–Littlewood–Sobolev–Stein–Weiss conditions. As auxiliary results, we give necessary and sufficient conditions for the boundedness of the Hardy and Bellman operators in Lebesgue spaces with Dunkl weight and piecewise-power weights.

This is a joint research with D.V. Gorbachev (TSU, Tula) and S.Yu. Tikhonov (CRM, Barcelona).

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The regularity of the solutions of the abstract evolution equations of hyperbolic type

Yoritaka Iwata

The regularity of abstract evolution is discussed. Much attention is paid to the pseudo maximal regularity in the hyperbolic evolution equations [Kat70, Kat73]. In this paper, starting with explaining the concept of pseudo maximal regularity, a property similar to the maximal regularity effect is introduced based on the logarithmic representation for abstract evolution equations of hyperbolic type [Iwa17b]. While several applications have been proposed for the logarithmic representation [Iwa19a, Iwa17a, Iwa19b, Iwa19c] in terms of nonlinearity, its underlying algebraic property [Iwa18] is prominent. In conclusion, the properties inherent to the analytic semigroup is extracted within the framework of the hyperbolic type evolution equations.

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Rubio de Francia type extrapolation for grand Lebesgue spaces

Pankaj Jain

The celebrated extrapolation result of Rubio de Francia, in a generalized form, can be stated as follows: If (f, g) is a pair of non-negative measurable functions such that for some $1 \leq p_0 < \infty$, the inequality

$$\int_{\mathbb{R}^n} f^{p_0}(x)w(x)dx \leq C \int_{\mathbb{R}^n} g^{p_0}(x)w(x)dx$$

holds for every $w \in A_{p_0}$ (the Muckenhoupt class of weights), then for every $1 \leq p < \infty$, the inequality

$$\int_{\mathbb{R}^n} f^{p}(x)w(x)dx \leq C \int_{\mathbb{R}^n} g^{p}(x)w(x)dx$$

holds for every $w \in A_p$. In this talk, we shall discuss some very recent extrapolation results in the framework of grand Lebesgue spaces defined on sets having finite as well as infinite measure. Both diagonal as well as off diagonal cases will be considered.

Moving and oblique observations of beams and plates

Philippe Jaming

We study the observability of the one-dimensional Schrödinger equation and of the beam and plate equations by moving or oblique observations. Applying different versions and adaptations of Ingham's theorem on nonharmonic Fourier series, we obtain various observability and non-observability theorems. Several open problems are also formulated at the end of the paper.

This is joint work with V. Komornik

On self-intersections of Laurent polynomials

Sergei Kalmykov

In this talk we discuss a bound on the number of self-intersection of curves with polynomial parameterization to the case of Laurent polynomials

$$p(z) = \sum_{k=m}^{n} a_k z^k, \quad z \in \mathbb{C} \setminus \{0\}, \tag{4}$$

where $m, n \in \mathbb{Z}$, $a_m \neq 0$, and $a_n \neq 0$. Our results generalize Quine's estimate [Qui73]. As an application, for example, we show that circle embeddings are dense among all maps from a circle to a plane with respect to an integral norm. And, in particular, it follows that the Fourier coefficients $\hat{f}$ of a circle embedding $f : \mathbb{T} \to \mathbb{C}$ can be arbitrarily close to any element of $\ell^2(\mathbb{Z})$.

This is based on a joint work with L.V. Kovalev [KK] and supported by Russian Foundation for Basic Research (grant 18-31-00101).

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The Vitali convergence theorem of nonlinear integral functionals

Jun Kawabe

The Choquet [Cho54, Sch86], Šipoš [Š79], Sugeno [RA80, Sug74], and Shilkret [Shi71] integrals of a nonnegative measurable function with respect to a nonadditive measure (also called a monotone measure, a fuzzy measure, or a capacity) may be considered as nonlinear integral functionals defined on the product of the space of all nonadditive measures and the space of all nonnegative measurable functions. They are widely used in application areas such as decision theory under uncertainty, game theory, data mining and others. For those functionals, their continuity corresponds to some convergence theorems of integrals, which mean that the limit of the integrals of a sequence of functions is the integral of the limit function. Thus many attempts have been made to formulate the counterparts of the monotone, the bounded, and the dominated convergence theorems of the Choquet, Šipoš, Sugeno, and Shilkret integrals.

The purpose of this talk is to present the Vitali convergence theorem of such nonlinear integrals in a unified way by formulating it in terms of nonlinear integral functionals with some properties. A key ingredient is a perturbation of nonlinear integrals that manages the small change of the integral value arising as a result of adding small amounts to a measure and an integrand [Kaw15, Kaw16, Kaw19].

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Best proximity point of contraction type mapping in metric space

Kyung Soo Kim

The purpose of this article, we consider the existence of a unique best proximity point $x^* \in A$ such that $d(x^*, Tx^*) = \text{dist}(A, B)$ for generalized φ-weak contraction mapping $T : A \to B$, where $A, B(\neq \emptyset)$ are subsets of a metric space (X, d).

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A moment inequalities for stochastic integral and applied in the stochastic differential equation

Young-Ho Kim

In this presentation, we shall apply Itô's formula deal with several important moment inequalities for stochastic integrals as well as the Gronwall-type inequalities. The moment inequalities of stochastic integral have been widely applied in the theory of ordinary differential equations and stochastic differential equations to prove the results on existence, uniqueness, boundedness, comparison, continuous dependence, perturbation and stability etc. For the moment inequality relevant to this topic, we first introduce previous results on stochastic integral inequalities. Next, we deal with some application for the solutions of the stochastic differential equation under weakened Hölder condition, a weakened linear growth condition, and a contractive condition.

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On the Lebesgue constants for convex polyhedra

Yurii Kolomoitsev

The talk is devoted to the Lebesgue constants for polyhedral partial sums of Fourier series. Upper and lower estimates of the Lebesgue constant for general convex polyhedra will be presented. In the two-dimensional case, we will give also new asymptotic formulas for the Lebesgue constants of anisotropic dilations of a triangle and rhombus.

This is a joint research with T. Lomako (IAMM NAS of Ukraine).

This research was partially supported by DFG project KO 5804/1-1.

Ingham type theorems and some applications to observability problems

Vilmos Komornik

We report on some generalizations of Ingham's classical extension of Parseval's equality, and we illustrate their usefulness to solve some observability problems related to various vibrating systems, including linear plate models and systems of strings and beams.

Most of the results presented in the talk have been obtained in collaboration with C. Baiocchi, P. Loreti, M. Mehrenberger, B. Miara and A.C. Lai.

On subsequences of almost everywhere convergence of partial trigonometric Fourier sums

Sergey V. Konyagin

Let $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$, $L(\mathbb{T})$ be the set of all integrable functions $f : \mathbb{T} \to \mathbb{C}$. We associate with a function $f \in L(\mathbb{T})$ its trigonometric Fourier series

$$f \sim \sum_{k=-\infty}^{\infty} \hat{f}(k)e^{ikx}, \quad \hat{f}(k) = \frac{1}{2\pi}\int_{\mathbb{T}} f(x)e^{-ikx}dx.$$

For $n \in \mathbb{N}$ define the n-th partial sum of f as

$$S_n(f;x) = \sum_{k=-n}^{n} \hat{f}_k e^{ikx}.$$

Let $\exp^*(0) = 1$ and $\exp^*(r) = \exp(\exp^*(r-1))$ for a positive integer r. Recall that a sequence of positive integers $\{n_j\}_{j\in\mathbb{N}}$ is lacunary if there exists a number $\rho > 1$ such that $n_{j+1}/n_j \geq \rho$ for any $j \in \mathbb{N}$.

Theorem 1. *There exists a real function $f \in L(\mathbb{T})$ such that for any almost everywhere convergent subsequence of partial sums $\{S_{n_j}(f)\}$ with a lacunary sequence $\{n_j\}$ we have*

$$n_j \geq \exp^*\left((\log\log j)^{1-o(1)}\right) \quad (j \to \infty).$$

Delay differential Painlevé equations and Nevanlinna theory

Risto Korhonen

One way in which difference Painlevé equations arise is in the study of difference equations admitting meromorphic solutions of slow growth in the sense of Nevanlinna theory. The idea that the existence of sufficiently many finite-order meromorphic solutions could be considered as a version of the Painlevé property for difference equations was first advocated in [AHH00]. This is a very restrictive property, as demonstrated by the short list of possible equations obtained in [HK07] of the form $w(z+1) + w(z-1) = R(z, w(z))$, where R is rational in w with meromorphic coefficients in z, and w is assumed to have finite order but to grow faster than the coefficients. In this talk necessary conditions are obtained for certain types of rational delay differential equations to admit a non-rational meromorphic solution of hyper-order less than one. The equations obtained include delay Painlevé equations and equations solved by elliptic functions [HK17].

This is a joint research with Rod Halburd (University College London).

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Weierstrass type density for weighted multivariate polynomials

András Kroó

In this talk we will consider weighted polynomial approximation on *unbounded multidimensional domains* in the spirit of the weighted version of the Weierstrass trigonometric approximation theorem according to which every continuous function on the real line with equal finite limits at $\pm\infty$ is a uniform limit on $\mathbb{R}$ of weighted algebraic polynomials $(1+t^2)^{-n}p_{2n}$. A similar statement in the multivariate setting will be shown to hold for weighted polynomials of the form $w^n(\mathbf{x})p_n(\mathbf{x})$ and a general class of positive *convex* weights. In addition, some typical *non convex* weights $w_\alpha(\mathbf{x}) := (1+|x_1|^\alpha+...+|x_d|^\alpha)^{\frac{1}{\alpha}}, 0<\alpha<1$ will be also considered. In this case in order for weighted polynomial approximation to hold it is necessary that the function vanishes on a certain exceptional set consisting of all coordinate hyper planes and ∞. Moreover, in case of rational α this condition turns out to be also sufficient.

Furthermore, we consider similar weighted multivariate approximation by weighted polynomials of the form $w^{\gamma_n}(\mathbf{x})p_n(\mathbf{x})$ where p_n is a multivariate polynomial of degree at most n, w is a given nonnegative weight with nonempty zero set, and $\gamma_n = o(n)$. We study the question if every continuous function vanishing on the zero set of w is a uniform limit of these weighted polynomials. It turns out that for various classes of weights in order for this approximation property to hold it is necessary and sufficient that $\gamma_n = o(n)$.

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Finite sums of ridge functions on convex subsets of $\mathbb{R}^n$

Alexander Kuleshov

We assume that $n \geq 2$ and $E \subset \mathbb{R}^n$ is a set. A closed convex set E with a non-empty interior is called a *convex body.* A *ridge function* on E is a function of the form $\varphi(\mathbf{a}\cdot\mathbf{x})$, where $\mathbf{x} = (x_1,\ldots,x_n) \in E$, $\mathbf{a} = (a_1,\ldots,a_n) \in \mathbb{R}^n \setminus \{\mathbf{0}\}$, $\mathbf{a}\cdot\mathbf{x} = \sum_{j=1}^n a_jx_j$ and φ is a real-valued function defined on $\Delta(\mathbf{a}) = \{\mathbf{a}\cdot\mathbf{x} : \mathbf{x} \in E\}$. On a set E, consider a sum of ridge functions

$$f(\mathbf{x}) = \sum_{i=1}^m \varphi_i(\mathbf{a}^i\cdot\mathbf{x}).$$

Let E be a convex body. We study the smoothness properties of the functions φ_i under certain assumptions on smoothness of f. We prove that the continuity of f implies that every φ_i belongs to the VMO space on every compact interval of its domain. Also, we prove that for the existence of finite limits of the functions φ_i at the corresponding boundary points of their domains, it suffices to assume the Dini condition on the modulus of continuity of f at some boundary point of E. Also we prove that the obtained (Dini) condition is sharp. Then we extend some of the results on the $C^k(E)$ classes for $k \geq 1$. We prove that for measurable functions φ_i the general implication $f \in C^k(E) \Rightarrow \varphi_i \in C^k(\Delta(\mathbf{a}^i))$ $(i = 1,\ldots,m)$ holds iff the boundary of E is *smooth.*

(Dense) graph limit theory: from combinatorics to analysis, and back again

Dávid Kunszenti-Kovács

Graph limit theory, motivated by questions in combinatorics, network theory and statistical physics, has become a significant topic over the past decade. The talk aims to present some of the developments in this area, mainly in the dense graph case, highlighting how discrete questions can turn into interesting problems in analysis, and how the latter can serve as a unifying language for a wide range of combinatorial questions.

Joint work with Á. Backhausz, L. Lovász and B. Szegedy.

Spectral synthesis on discrete Abelian groups

Miklós Laczkovich

In questions of spectral synthesis on Abelian topological groups the classes of polynomials, exponential polynomials, generalized polynomials and generalized exponential polynomials play the most important roles. We discuss the characterization of these classes, give a survey of the variants of spectral synthesis and spectral analysis on discrete Abelian groups, outline their algebraic background, and mention some open questions.

Szegő's triangle of orthogonal polynomials and related extremal problems

Alexey Lukashov

In 1964 Gábor Szegő considered triangle which consists of three types of orthogonal polynomials: on the real line, on the unit circle, and trigonometric (by)orthogonal.

We present a survey of related extremal problems of the approximation theory. In particular, inequalities for polynomials and rational functions, as well as Lebesgue constants for interpolation on several intervals (arcs) are discussed.

On the global sup-norm of spherical cusp forms on $\mathrm{PGL}(n)$

Péter Maga

Estimating automorphic forms of arithmetic quotients in terms of their Laplacian eigenvalue is an interesting question in analytic number theory on higher-rank groups. Recent results show that there is an essential difference between compact and non-compact domains (at least, for the general linear group). In the talk, I give a summary of these results and present some new ones for spherical cusp forms on $\mathrm{PGL}(n,\mathbb{Z})\backslash\mathrm{PGL}(n,\mathbb{R})$.

Joint results with Valentin Blomer and Gergely Harcos.

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Fuglede's conjecture for convex bodies

Máté Matolcsi

A set $\Omega \subset \mathbb{R}^d$ is said to be spectral if the space $L^2(\Omega)$ has an orthogonal basis of exponential functions. A conjecture due to Fuglede (1974) stated that $\Omega \subset \mathbb{R}^d$ is a spectral set if and only if it can tile the space by translations. While this conjecture was disproved for general sets, it has long been known that if a *convex body* $\Omega \subset \mathbb{R}^d$ tiles the space by translations then it indeed must be spectral. The "spectral implies tiling" direction of the conjecture for convex bodies, however, was proved only in $\mathbb{R}2$, and also in $\mathbb{R}^3$ under the a priori assumption that Ω is a convex polytope. In higher dimensions, this direction of the conjecture remained completely open for convex domains (even in the case when Ω is a convex polytope), and could not be treated using the previously developed techniques.

Recently, in a joint work with Nir Lev, we have settled Fuglede's conjecture for convex bodies in the affirmative in all dimensions, i.e. we proved that if a convex body $\Omega \subset \mathbb{R}^d$ is spectral then it can tile the space by translations. Our approach involves a construction from crystallographic diffraction theory, that allows us to establish a new geometric "weak tiling" condition which is necessary for the spectrality of Ω.

A Nadler type result for iterated multifunction systems

Radu Miculescu

A very famous generalization of Banach's contraction principle to the framework of set-valued functions is due to Markin (see [Mar68]) and Nadler (see [Nad69]).

One of the main tools used to model self-similarity is the concept of iterated function system introduced by J. Hutchinson (see [Hut81]). Iterated multifunction systems generalize IFSs from the standard point-to-point to set-valued contractions.

By combining the above two mentioned lines of research, we prove a Nadler type result for iterated multifunction systems. Our result is a generalization of Nadler's fixed point theorem. We also emphasize the connection between our result and the Kameyama's concept of topological self-similar system (see [Kam00]).

This is a joint research with Alexandru Mihail (Bucharest University, Romania).

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On a “martingale property” of series by general Franklin systems

Vazgen Mikayelyan

The general Franklin system corresponding to a given dense sequence of different points $\mathcal{T} = (t_n, n \geq 0)$ in $[0,1]$ is a sequence of orthonormal piecewise linear functions with knots from $\mathcal{T}$, that is, the nth function from the system has knots $t_0, \ldots, t_n$. It is known that the general Franklin system is a basis in $C[0,1]$ and in $L^p[0,1]$ for $1 < p < \infty$ (see [Cie63]), and it is an unconditional basis in $L^p[0,1]$ for $1 < p < \infty$ (see [GK04]).

We proved that if $\{f_n\}_{n=0}^{\infty}$ is the general Franklin system corresponding to a quasi-dyadic and strongly regular sequence $\mathcal{T}$ (see [Gev13]) and $\sigma_n(x) = \sum_{k=0}^{n} a_k f_k(x)$, then

$$\operatorname{mes}\left\{x \in [0,1] : -\infty < \liminf_{n\to\infty} \sigma_n(x) \leq \limsup_{n\to\infty} \sigma_n(x) = +\infty\right\} = 0.$$

This property was proved for classical Franklin system in [Gev19] by Gevorkyan, which was an analogue of Konyagin's theorem for trigonometric series (see [Kon88]), which was an answer to Lusin's famous problem on convergence of trigonometric series to $+\infty$.

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On the Cesaro summability of negative order of double series with respect to block-orthonormal systems

Givi Nadibaidze

Below a question connected to the almost everywhere summability by $(C, -1 < \alpha < 0, -1 < \beta < 0)$ methods of double series with respect to block-orthonormal systems are considered.

Let $\{M_p\}$ and $\{N_q\}$ be the increasing sequences of natural numbers and $\Delta_{p,q} = (M_p, M_{p+1}] \times (N_q, N_{q+1}]$, $(p, q \geq 1)$. Let $\{\varphi_{mn}\}$ be a system of functions from $L^2((0,1)^2)$. The system $\{\varphi_{mn}\}$ will be called a $\Delta_{p,q}$-orthonormal system ($\Delta_{p,q}-$ONS) if:

1) $\|\varphi_{mn}\|_2 = 1$, $m = 1, 2, \ldots, n = 1, 2, \ldots$;
2) $(\varphi_{ij}, \varphi_{kl}) = 0$, for $(i,j), (k,l) \in \Delta_{p,q}$, $(i,j) \neq (k,l)$, $(p, q \geq 1)$.

Statements connected with the $(C, -1 < \alpha < 0, -1 < \beta < 0)$ almost everywhere summability of series

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\varphi_{mn}(x,y)$$

with respect to $\Delta_{p,q}$-orthonormal system $\{\varphi_{mn}\}$ be given. In particular, it is stated the conditions on the sequences $\{M_p\}$ and $\{N_q\}$, when the condition

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^2 m^{-2\alpha} n^{-2\beta} < \infty,$$

guarantees the $(C, -1 < \alpha < 0, -1 < \beta < 0)$ almost everywhere summability of corresponding block-orthogonal series. On the other hand the multiplier $m^{-2\alpha}n^{-2\beta}$ is unimprovable.

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An electrostatic extremal problem on the circle and on the line

Zsuzsanna Nagy-Csiha

In the following, we investigate a problem motivated by a question raised by Pap and Schipp in papers [PS01] and [PS15], connected to the Malmquist–Takenaka system.

Let $\mathbf{a}$ be a complex sequence, $\mathbf{a}: \ a_1, \ldots, a_n \in \mathbb{D}$ and define the Blaschke product as

$$B(z) = B_{\mathbf{a}}(z) := \chi \prod_{j=1}^{n} \frac{z - a_j}{1 - \overline{a_j} z}, \quad |\chi| = 1.$$

For given $a_1, \ldots, a_n \in \mathbb{D}$, we consider the critical points of the Blaschke product. It is known (see e.g. [SW19]) that $B'(z) = 0$ has $2n-2$ solutions: $n-1$ of them are in the unit disk, and if ζ is a critical point, then $\zeta^* := 1/\overline{\zeta}$ is also a critical point with the same multiplicity.

Let $\chi = 1$. For given $\delta \in \mathbb{R}$, we consider $T(\delta)$, the solution curve of $B(z) = e^{i\delta}$:

$$T(\delta) := \left\{(\tau_1, \ldots, \tau_n) \in \mathbb{R}^n : \ B\left(e^{i\tau_j}\right) = e^{i\delta}, \ j = 1, \ldots, n, \ \tau_1 < \ldots < \tau_n\right\}. \tag{5}$$

We introduce the following electrostatic problem. Place $n-1$ pairs of fix protons with $1/2$ weight to the critical points ζ_k, ζ_k^*, $k = 1, \ldots, n-1$ and n freely moving negative unit charge to $w_1, ..., w_n \in \mathbb{C}$, $|w_j| = 1$, $j = 1, \ldots, n$ points of the unit circle. The discrete energy of this electron configuration can be described with the following function:

$$W(w_1, ..., w_n) = \sum_{k=1}^{n-1}\sum_{j=1}^{n} \log|(w_j - \zeta_k)(w_j - \zeta_k^*)| - 2 \sum_{1 \leq j < k \leq n} \log|w_j - w_k|.$$

Our goal is to show that for every $\delta \in \mathbb{R}$ the $\widetilde{W}(\tau_1, \ldots, \tau_n) := W(w_1, \ldots, w_n)$ real valued function has global minimum in $T(\delta)$ (5).

We use the following general form of Cayley transform as an important tool for transforming the problem onto the real line:

$$C(z) = C_\theta(z) := i\frac{1 + ze^{-i\theta}}{1 - ze^{-i\theta}}, \quad \theta \in \mathbb{R}.$$

By using the property $C_\theta(e^{i\theta}) = \infty$ we can use the result of Semmler and Wegert [SW19] for the transformed problem, and we can complete the proof using the result of Pap and Schipp [PS01], [PS15]. Finally, we can answer the question asked by Pap and Schipp.

This is a joint research with Béla Nagy (SZTE), Marcell Gaál (RI) and Szilárd Révész (RI).

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On the Borel summability of WKB solutions near a simple pole

Gergő Nemes

We consider the following Schrödinger-type differential equation:

$$\frac{\mathrm{d}^2 w(u,z)}{\mathrm{d}z^2} = (u^2 f(z) + g(z))w(u,z),$$

where u is a large positive parameter, and $f(z)$ and $g(z)$ are analytic functions of z apart from countably many singularities. We assume that $f(z)$ has a simple pole, and $g(z)$ has at most a double pole at the origin. The Liouville transformation $\xi = \int_0^z \sqrt{f(t)}\mathrm{d}t$ brings the equation into the standard form

$$\frac{\mathrm{d}^2 W(u,\xi)}{\mathrm{d}\xi^2} = (u^2 + \psi(\xi))W(u,\xi), \quad W(u,\xi) = f^{1/4}(z)w(u,z).$$

The singularities of $f(z)$ and $g(z)$ at $z = 0$ are mapped into a double pole of the function $\psi(\xi)$ at $\xi = 0$. It is known that this equation has formal solutions of the form

$$W_{1,2}(u,\xi) = \mathrm{e}^{\pm\xi u} \sum_{n=0}^{\infty} \frac{\mathsf{A}_n(\pm\xi)}{u^n}.$$

These are called the WKB solutions. We study the Borel summability of these WKB solutions near the double pole of the function $\psi(\xi)$ at the origin. It is shown that both of the formal series are Borel summable in every closed strip $\{\xi : |\Im\xi| \le \gamma\}$ contained in the domain of analyticity of $\psi(\xi)$ apart from the Stokes rays $\arg\xi = 0,\ 2\pi$ and $\arg\xi = \pm\pi$ emanating from the origin. We determine the type of singularities of the Borel transforms near the origin when $|\xi|$ is small and also provide global connection formulae between the solutions $W_1(u,\xi)$ and $W_2(u,\xi)$. Possible extensions to other types of singularities and to more general equations will also be discussed.

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Maximal convergence of Faber series in Smirnov-Orlicz classes

Burçin Oktay Yönet

Let G be a simply connected domain in the complex plane $\mathbb{C}$ bounded by a curve which belongs to a special subclass of smooth curves and let f be a function which is analytic in the canonical domain $G_R \supset G$, $R > 1$. In this case, we study the remainder term

$$R_n(z,f) = f(z) - \sum_{k=0}^{n} a_k \varphi_k(z) = \sum_{k=n+1}^{\infty} a_k \varphi_k(z), \quad z \in \Gamma = \partial G,$$

where a_k, $k = 0,1,2,\ldots$ are the Faber coefficients of the function f with respect to the domain G and $\varphi_k(z)$ is the Faber polynomial of order k for the domain G. We obtain results on the maximal convergence of the Faber series expansions of the function f which belongs to the Smirnov-Orlicz class $E_M(G_R)$, $R > 1$.

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Symmetrization inequalities for probability metric spaces with convex isoperimetric profile

Walter A. Ortiz

We obtain symmetrization inequalities on probability metric spaces that admit a convex isoperimetric estimator which incorporate in their formulation the isoperimetric estimator and that can be applied to provide a unified treatment of sharp Sobolev-Poincaré and Nash type inequalities.

This is a joint research with Joaquim Martín (Universitat Autónoma de Barcelona).

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An approximate formula for the binary Goldbach's problem

János Pintz

In the lecture we sketch an approximate formula for the contribution of the major arcs in Goldbach's problem as a function of a relatively small number of zeros of Dirichlet L-functions, lying extreme near to the boundary line $\Re s = 1$. This formula plays a crucial role in the proof of the following approximate forms of the Goldbach conjecture.

By the use of other ideas and methods we can show, for example, the following approximate forms of the binary Goldbach conjecture. Concerning the size of the exceptional set in Goldbach's problem we can show:

Theorem 1 (J.P.). *The number of even integers up to N which are not representable as the sum of two primes is at most $O(N^{0.72})$.*

We can improve the earlier results in the unconditional case of the Linnik-Goldbach problem:

Theorem 2 (J.P. and I.Z. Ruzsa). *Every sufficiently large even integer can be written as the sum of two primes and eight powers of* 2.

We can give a stronger estimate for the number of Goldbach exceptional numbers in polynomial sequences, than the one proved by Brüdern, Kawada and Perelli. We just mention the result in case if the even integer to be represented as the sum of two primes is the double of a square.

Theorem 3 (A. Perelli and J.P.). *Let $c > 4/5$ be fixed. The number of even integers of the form $2n^2$ which are not representable as the sum of two primes is for $n < N$ at most $O(N^c)$.*

The Prékopa-Leindler inequality and sumsets

Imre Z. Ruzsa

Approximation of the step–function, Kolmogorov width of W_1^1 and approximate rank

Konstantin S. Ryutin

In this talk we plan to discuss the notion of an approximate rank of a matrix and its relations with a more familiar for analysts notion of a Kolmogorov width. We give necessary definitions. For a matrix $A = (A_{i,j})$ and $\varepsilon > 0$ let $\mathrm{rank}_\varepsilon(A) := \min\{\mathrm{rank}B\colon \max_{i,j}|A_{i,j} - B_{i,j}| \le \varepsilon\}$. This notion is actively studied in combinatorics and computer science. For X a linear normed space; $W, L \subset X$ we define the deviation of W from L as $E(W,L)_X := \sup_{x\in W}\inf_{y\in L}\|x-y\|_X$ and the *Kolmogorov n-width* of a set W in X as $d_n(W,X) := \inf_{\substack{L_n\subset X\\ \dim L_n\le n}} E(W,L_n)_X$, where the inf is taken over all linear subspaces of X with dim $\le n$.

The main focus of our research is the problem of the order for $d_n(W_1^1, L^q[0,1]), 2 < q < \infty$ (the Kolmogorov width for the convex hull of the step-functions). We show that it is closely related with the order of $\mathrm{rank}_\varepsilon$ for a specific upper-triangular matrix (with 1 on and above the diagonal) and we try to make sharp estimates for this quantity. This problem can be approached with different techniques from harmonic analysis, approximation theory, probability. In this talk we plan to describe these approaches and the results that can be obtained.

This is a joint research with B.S. Kashin and Yu.V. Malykhin (Steklov MI, MSU, Moscow).

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Chebyshev polynomials on circular arcs

Klaus Schiefermayr

In this talk, we give an explicit representation of the Chebyshev polynomial on a given circular arc

$$A_\alpha := \big\{z \in \mathbb{C}:\ |z| = 1, -\alpha \le \arg(z) \le \alpha\big\}, \qquad 0 < \alpha \le \pi,$$

(a problem which was first considered in [TD91]), which is done in two steps: In the first step, following [PS02], we give an explicit representation of the Chebyshev polynomial (of degree N) on A_α in terms of the Chebyshev polynomial with respect to the weight function $w(x) := 1$ (for N even) and $w(x) := \sqrt{1-x^2}$ (for N odd) on the two real intervals $[-1,-a] \cup [a,1]$, where $a := \cos(\frac{\alpha}{2})$. For this representation, we will need the mapping $z \mapsto \frac{1}{2}\left(\sqrt{z} + \frac{1}{\sqrt{z}}\right)$ which maps $\{z \in \mathbb{C} : |z| = 1, \Im\{z\} \geq 0\}$ bijectively onto the interval $[0, 1]$. In the second step, these Chebyshev polynomials (with respect to $w(x) := 1$ and $w(x) := \sqrt{1 - x^2}$) are represented with the help of Jacobian elliptic and theta functions. These representations go back to [Akh28] and [Kru61]. The talk is based on the paper [Sch19].

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The valence of harmonic mappings in the plane

Olivier Sète

For a harmonic mapping f (a complex-valued function with $\Delta f = 0$) and given $\eta \in \mathbb{C}$, we study the number of solutions of

$$f(z) = \eta, \tag{6}$$

as well as the valence of f. Harmonic mappings, and in particular harmonic polynomials and rational harmonic functions, have been studied by Sheil-Small, Bshouty, Lyzzaik, Wilmshurst, Khavinson, Neumann, Geyer, and others, but little is known about the valence in general.

We derive a formula for counting the number of solutions of (6) for harmonic mappings defined in $\mathbb{C}$, with the possible exception of finitely many poles. The formula depends on the behaviour of f near its poles and near infinity, and on the position of η relative to the caustics of f, i.e., the image of the critical set. A key ingredient is the (topological) argument principle for complex-valued continuous functions. Moreover, we discuss the effect of a change in η on the number of solutions, as well as their positions.

This is joint research with Jan Zur (TU Berlin).

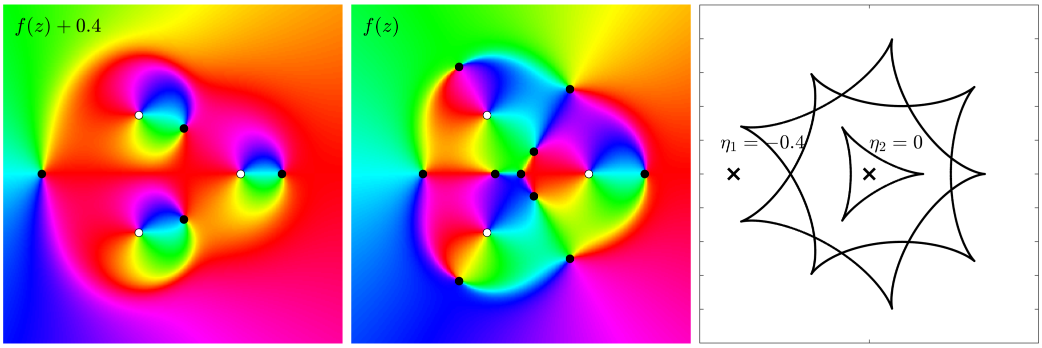

Figure 1: Phase plots of $f(z) - \eta$ for $f(z) = z - \overline{z^2/(z^3 - 0.6^3)}$ and $\eta_1 = -0.4$ (left) and $\eta_2 = 0$ (middle). The right plot shows the caustics and η_1, η_2 in the η-plane.

Dynamical Cantor sets and quasiconformal mappings

Hiroshige Shiga

In the theory of Teichmüller space of Riemann surfaces, we consider the set of Riemann surfaces which are quasiconformally equivalent. Here, we say that two Riemann surfaces are quasiconformally equivalent if there is a quasiconformal homeomorphism between them. Hence, at the first stage of the theory, we have to know a condition for Riemann surfaces to be quasiconformally equivalent.

The condition is quite obvious if the Riemann surfaces are topologically finite. On the other hand, for Riemann surfaces of topologically infinite type, the situation is rather difficult.

In this talk, we discuss the quasiconformal equivalence of regions which are complements of some Cantor sets, i.e., the limit sets of some Kleinian groups, the Julia sets of some rational functions and random Cantor sets. Moreover, we may estimate the Teichmüller distances between regions defined by random Cantor sets in terms of randomness for those Cantor sets.

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Congruences and exponential sums over multiplicative subgroups in finite fields

Iurii Shteinikov

Let $\mathbb{Z}_p$ be finite field with p elements and let $G \subset \mathbb{Z}_p^*$ be multiplicative subgroup. Consider the following exponential sums

$$S(a, G) = \sum_{g \in G} e^{2\pi i \frac{ag}{p}}$$

We are interested in non-trivial estimates of the form $|S(a, G)| = o(|G|)$. Upper estimates for $|S(a, G)|$ can be obtained from upper estimates for $T_k(G)$, where

$$T_k(G) = |\{(x_1, \dots, x_2) \in G^{2k} : x_1 + \dots + x_k = x_{k+1} + \dots + x_{2k}\}|$$

Several non-trivial estimates for T_k were obtained in using method due to S.A. Stepanov. In my talk I am planning to present the recent estimate for $T_3(G)$ and some other applications.

This is a joint research with B. Murphy (Heilbronn Institute for Mathematical Research), I. Shkredov (Steklov Mathematical Institute) and M. Rudnev (Bristol University).

On the Voice transform induced by a representation of the dyadic and 2-adic Blaschke group

Ilona Simon

In this talk we consider the dyadic and 2-adic Blaschke groups which are generated by the composition of 2-adic and dyadic Blaschke functions, some linear fractional transformations which preserve the unit circle and the unit disc. Here we investigate the voice transform induced by a representation of the Blaschke group over the proper Hardy space.

This is a joint research with Margit Pap (University of Pécs).

Two-sided estimates for the sums of a sine series with convex slowly varying coefficients

Aleksei P. Solodov

Let's consider the sum of a sine series $g(\mathbf{b}, x) = \sum_{k=1}^{\infty} b_k \sin kx$. It is known that the sum of a sine series with convex coefficients $\mathbf{b} = \{b_k\}_{k \in \mathbb{N}}$ is positive in the interval $x \in (0, \pi)$. To estimate values of the sum near the origin traditionally was used function introduced by Salem $v(\mathbf{b}, x) = x \sum_{k=1}^{m(x)} k b_k$, $m(x) = [\pi/x]$. If the sequence of coefficients $\mathbf{b}$ is slowly varying, then the following asymptotic formula holds

$$\frac{2}{\pi^2} v(\mathbf{b}, x) \sim \frac{b_{m(x)}}{x}, \qquad x \to +0.$$

Aljančić, Bojanić and Tomić established [ABT56], that for any convex slowly varying null sequence $\mathbf{b}$ the following asymptotic formula holds

$$g(\mathbf{b}, x) \sim \frac{b_{m(x)}}{x}, \qquad x \to 0.$$

Telyakovskiĭ showed [Tel95], that the difference $g(\mathbf{b}, x) - b_{m(x)}/x$ in order is comparable with the function

$$\sigma(\mathbf{b}, x) = x \sum_{k=1}^{m(x)-1} \frac{k(k+1)}{2} \Delta b_k, \quad \Delta b_k = b_k - b_{k+1}.$$

We refine marked result. A two-sided estimate of this difference with sharp constants is obtained.

Theorem. *For any convex slowly varying null sequence* $\mathbf{b}$ *and* $x \in (0, \pi/11)$,

$$\frac{6(\pi - 1)}{\pi^3} \sigma(\mathbf{b}, x) - \frac{\Delta b_{m(x)}}{\pi} < g(\mathbf{b}, x) - \frac{b_{m(x)}}{x} < \sigma(\mathbf{b}, x).$$

There are convex slowly varying null sequences $\underline{\mathbf{b}}$, $\overline{\mathbf{b}}$, *such that*

$$\varliminf_{x \to +0} \left(g(\underline{\mathbf{b}}, x) - \frac{b_{m(x)}}{x} \right) / \sigma(\underline{\mathbf{b}}, x) = \frac{6(\pi - 1)}{\pi^3},$$
$$\varlimsup_{x \to +0} \left(g(\overline{\mathbf{b}}, x) - \frac{b_{m(x)}}{x} \right) / \sigma(\overline{\mathbf{b}}, x) = 1.$$

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Characterization of associate function spaces and principle of duality

Vladimir D. Stepanov

We analyse the problem of characterization of function spaces associated to a given function spaces or cones. The situation is rather different for an ideal, that is with lattice property, and non-ideal function spaces. Namely, the notion of associate norm bifurcates for a non-ideal space. We provide several examples of such a characterization including the weighted Sobolev space of the first order on the real line. The talk is based on the publications cited below.

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On Pecherskij-Révész's theorem

Vaja Tarieladze

Dedicated to 80-th birthday anniversary of Zaur Chanturia (1939-1989)

In 1905 H. Lebesgue has shown that the Fourier series of a continuous periodic function f may not converge uniformly even if it converges point-wise to f. For some known conditions for uniform convergence see [Čan76]. In 1964 B.S. Stechkin [Ste62] has found a continuous periodic function f, whose Fourier series diverges at some points, but the rearranged Fourier series converges uniformly to f. Later D.V. Pecherskij [Pec88] and Sz.Gy. Révész [Rév94] independently have found an additional condition on the Fourier series of a continuous 2π-periodic function $f : \mathbb{R} \to \mathbb{C}$, which guarantees that some its rearrangement converges uniformly to f. We plan to discuss:

- the question whether a uniformly convergent Fourier series of a continuous 2π-periodic function $f : \mathbb{R} \to \mathbb{R}$ necessarily satisfies the Pecherskij-Révész's condition;
- the question whether an analog of Pecherskij-Révész's result remains true for a continuous 2π-periodic function $f : \mathbb{R} \to X$, where X is (real or complex) infinite-dimensional Banach space.

The talk is based mainly on [CGT12].

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Smoothness conditions in approximation theory

Sergey Tikhonov

We discuss new relationships between smoothness of functions and smoothness of approximation processes.

Korevaar-Schoen's directional energy and Ambrosio's regular Lagrangian flows

Alexander Tyulenev

We develop Korevaar-Schoen's theory of directional energies for metric-valued Sobolev maps in the case of RCD source spaces; to do so we crucially rely on Ambrosio's concept of Regular Lagrangian Flow. Our review of Korevaar-Schoen's spaces brings new (even in the smooth category) insights on some aspects of the theory. We develop the notion of "differential of a map along a vector field" and establish the parallelogram identity for $CAT(0)$ targets.

On the sharp constant in Hardy inequalities for a class of weighted polynomials and for finite sequences

Rumen Uluchev

We denote by $\mathcal{P}_n$ the set of real-valued algebraic polynomials of degree at most n. For the class of weighted polynomials $\mathcal{H}_n := \{f : f(x) = e^{-x/2}\, p(x),\ p \in \mathcal{P}_n\}$ we investigate the best (i.e. the smallest possible) constant c_n in the continuous Hardy L_2 inequality

$$\int_0^\infty \left(\frac{1}{x}\int_0^x f(t)\,dt\right)^2 dx \le c_n \int_0^\infty f^2(x)\,dx, \qquad f \in \mathcal{H}_n.$$

Our main result is two-sided estimates for c_n of the form

$$4 - \frac{c}{\ln n} < c_n < 4 - \frac{c}{\ln^2 n},$$

where c is a positive constant. It confirms the expected $\lim\limits_{n\to\infty} c_n = 4$, showing however that the convergence speed is rather slow.

In addition, we consider a discrete Hardy inequality for finite sequences of real numbers $\{a_k\}_{k=1}^n$. We prove two-sided estimates for the sharp constant d_n in the inequality

$$\sum_{k=1}^n \left(\frac{1}{k}\sum_{j=1}^k a_j\right)^2 \le d_n \sum_{k=1}^n a_k^2,$$

which are essentially the same as above for c_n in the continuous case.

This is a joint research with Dimitar K. Dimitrov (State University of Sao Paulo UNESP, Brazil), Ivan Gadjev and Geno Nikolov (Sofia University St. Kliment Ohridski, Bulgaria).

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Commutators and Bounded Mean Oscillation

Brett Wick

We will discuss some recent results about commutators of certain Calderon-Zygmund operators and BMO spaces and how these generate bounded operators on Lebesgue spaces. Results on the Heisenberg group, pseudoconvex domains with C^2 boundary, and other examples will be explained.

This talk is based on joint collaborative work.

Generalizations for certain p-valent functions

Emel Yavuz

Let $\mathcal{A}_p$ be the class of functions $f(z)$ of the form

$$f(z) = z^p + a_{p+1}z^{p+1} + \cdots$$

which are analytic and p-valent in the open unit disk $\mathbb{U}$. For functions $f(z) \in \mathcal{A}_p$, we consider the subclass $\mathcal{M}_p(\alpha, j)$ of $\mathcal{A}_p$ consisting of functions $f(z)$ which satisfy some condition with $f^{(j)}(z), (j = 0, 1, 2, \cdots, p)$ and some real α $(\alpha > p - j)$. The object of the present paper is to derive some interesting properties of $f(z)$ concerning with the class $\mathcal{M}_p(\alpha, j)$ such as subordination, distortion and coefficient inequalities:

Theorem 1. *If a function $f(z) \in \mathcal{A}_p$ satisfies the subordination*

$$\frac{zf^{(j+1)}(z)}{f^{(j)}(z)} - (p-j) \prec \frac{2(p-j-\alpha)z}{1-z} \quad (z \in \mathbb{U})$$

for some real $\alpha(\alpha > p - j)$ and $j = 0, 1, 2, \cdots, p$ then $f(z)$ belongs to the class $\mathcal{M}_p(\alpha, j)$.

Theorem 2. *If $f(z)$ belongs to the class $\mathcal{M}_p(\alpha, j)$, then*

$$\operatorname{Re}\left(\frac{(p-j)!f^{(j)}(z)}{p!z^{p-j}}\right) < \frac{\beta+1}{2} \quad (z \in \mathbb{U}),$$

where

$$\beta = \frac{1 + 2(\alpha - p + j)}{1 - 2(\alpha - p + j)}.$$

Theorem 3. *If $f(z) \in \mathcal{A}_p$ satisfies*

$$\operatorname{Re}\left(\frac{zf^{(j+2)}(z)}{f^{(j+1)}(z)}\right) < -\frac{1}{2} \quad (z \in \mathbb{U})$$

for $j = 0, 1, 2, \cdots, p$ then

$$\left|\frac{zf^{(j+1)}(z)}{f^{(j)}(z)} - \frac{\alpha}{2}\right| < \frac{\alpha}{2} \quad (z \in \mathbb{U}),$$

where $\alpha > p - j$.

Theorem 4. *If $f(z)$ belongs to the class $\mathcal{M}_p(\alpha, j)$, then*

$$|z|^{p-j}(1-|z|)^{2(\alpha-p+j)} \leq |f^{(j)}(z)| \leq |z|^{p-j}(1+|z|)^{2(\alpha-p+j)}$$

for $z \in \mathbb{U}$. The equality holds true for $f(z)$ given by

$$f^{(j)}(z) = \frac{z^{p-j}}{(1-z)^{2(p-j-\alpha)}}.$$

Theorem 5. *If $f(z)$ belongs to the class $\mathcal{M}_p(\alpha, j)$, then we have*

$$|a_{p+k}| \leq \frac{p!(p+k-j)!}{k!(p+k)!(p-j)!} \prod_{m=0}^{k-1} \left(m + 2(\alpha + j - p)\right).$$

The equality holds true for a function $f(z)$ given by

$$f^{(j)}(z) = z^{p-j}(1-z)^{2(\alpha+j-p)}.$$

This is a joint research with Tuğba Daymaz (İstanbul Kültür University, Turkey) and Shigeyoshi Owa.

On exceptional points for the Lebesgue density theorem

László Zsidó

Let m_d denote the Lebesgue measure on $\mathbb{R}^d$, $B \subset \mathbb{R}^d$ a Borel set, and χ_B the characteristic function of B. For every norm $\|\cdot\|$ on $\mathbb{R}^d$ and every $x \in \mathbb{R}^d$, we denote

$$\begin{aligned} B_{\|\cdot\|}(x,r) &:= \{y \in \mathbb{R}^d : \|y-x\| \le r\}, \qquad r>0, \\ \underline{D}_{\|\cdot\|,B}(x) &:= \liminf_{0<r\to 0} \frac{m_d\left(B \cap B_{\|\cdot\|}(x,r)\right)}{m_d\left(B_{\|\cdot\|}(x,r)\right)}, \\ \overline{D}_{\|\cdot\|,B}(x) &:= \limsup_{0<r\to 0} \frac{m_d\left(B \cap B_{\|\cdot\|}(x,r)\right)}{m_d\left(B_{\|\cdot\|}(x,r)\right)}. \end{aligned}$$

By the Lebesgue density theorem we have for every norm $\|\cdot\|$ on $\mathbb{R}^d$ and m_d-almost every $x \in \mathbb{R}^d$:

$$\underline{D}_{\|\cdot\|,B}(x) = \overline{D}_{\|\cdot\|,B}(x) = \chi_B(x).$$

In particular, assuming that $m_d(B) > 0$ and $m_d(\mathbb{R}^d \setminus B) > 0$, the functions $\underline{D}_{\|\cdot\|,B}$ and $\overline{D}_{\|\cdot\|,B}$ take both values 1 and 0.

We prove that if $m_d(B) > 0$ and $m_d(\mathbb{R}^d \setminus B) > 0$, then there is a point $x \in \mathbb{R}^d$ such that

$$\underline{D}_{\|\cdot\|,B}(x) \le \frac{1}{2} \le \overline{D}_{\|\cdot\|,B}(x) \tag{*}$$

for every norm $\|\cdot\|$ on $\mathbb{R}^d$.

It was previously known that, denoting by $\|\cdot\|_2$ the Euclidean norm on $\mathbb{R}^d$, if we assume that $m_d(B) > 0, m_d(\mathbb{R}^d \setminus B) > 0$, and

$$\underline{D}_{\|\cdot\|_2,B}(x) = \overline{D}_{\|\cdot\|_2,B}(x) \text{ for every } x \in \mathbb{R}^d,$$

then we have

$$\underline{D}_{\|\cdot\|_2,B}(x) = \overline{D}_{\|\cdot\|_2,B}(x) = \frac{1}{2}$$

for some $x \in \mathbb{R}^d$, that is (*) with $\|\cdot\| = \|\cdot\|_2$ holds true for some $x \in \mathbb{R}^d$ (see Theorem 7.4 in [ACC19]).

This is a joint research with Roberto Peirone (Università di Roma "Tor Vergata").

References

[ACC19] Alessandro Andretta, Riccardo Camerlo, and Camillo Costantini. Lebesgue density and exceptional points. *Proc. Lond. Math. Soc. (3)*, 118(1):103–142, 2019. ISSN 0024-6115. URL https://doi.org/10.1112/plms.12177.